Nonstandard methods and additive combinatorics: effective estimates for Jin's theorem

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Some of the most important results in combinatorics of numbers relate purely quantitative notions of largeness, such as the asymptotic density, to notions of combinatorial structure. For example, the celebrated Szemerédi theorem on arithmetic progressions asserts that any set of integers of positive asymptotic density contains arbitrarily long arithmetic progressions. Another result of this type is Jin's theorem: the sumset of two sets of positive asymptotic density is piecewise k-syndetic for some natural number k, where a set of integer is called piecewise k-syndetic if the union of k translates of it contains arbitrarily long intervals. This result has been obtained in 2004 by Reling Jin with nonstandard methods. Recently, Mauro Di Nasso has proved, again with nonstandard methods, a quantitative version of this theorem: the sumset $A + B$ of two sets A, B of integers of positive asymptotic density is piecewise k-syndetic for some k not greater than the inverse of the product of the densities of A and B. In this talk, I will present the key ideas of the proof of this result and I will mention how these, with suitable modifications, can be used to obtain the same result in the broader setting of arbitrary amenable groups.